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AN EQUATION BALANCE FOR CLASS-ROOM USE.

By E. W. PONZER, Leland Stanford Junior University.

Of equation balances, many types of which have been described, the one which, in the writer's opinion, possesses the greatest merit for demonstration work in the class-room in connection with the numerical solution of equations is that of Lalanne, presented to the French Academy of Sciences in 1840.[1] The principle on which the balance works is that of moments—a principle fundamental to many machines designed to solve equations.

Perhaps it would be in order here to briefly review the application of the principle so that the mechanical methods by which it is carried out may be clearly understood. Given the equation

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n = 0$$

with real coefficients and x_1 as a root of the equation, we may consider each term to be in the form of a moment (positive or negative according to the sign of the coefficient, the root being assumed positive) where a quantity proportional to the coefficient takes the place of the weight and the moment arm is represented by one of the terms $x_1, x_1^2, x_1^3, \cdots x_1^n$. The moment is equal to zero only when x_1 is a root of the equation. A similar condition exists for any other real root of the equation. Imaginary roots would necessarily require a different treatment.

To make use of this principle in a mechanical way all that is necessary is to construct accurately the curves $y = \pm x$, $y = \pm x^2$, $y = \pm x^3$, $\cdots$ and then to use the ordinates corresponding to a real root of the equation as moment arms, together with suitable weights for the coefficients.

The figure on page 284 (showing the standard and right half of the balance) clearly illustrates the construction, which is entirely home-made. The axis on which the root is read is the x-axis. The curves up to $y = \pm x^4$ were plotted between $x = 0$ and $x = 1$ and the brass strips on which the weights are suspended were bent to shape and fastened as shown. The curves all pass through the origin and the points (1, 1) or (1, − 1). The guide moves these weights along the curves, keeping them at all times lined up parallel to the axis of the arms. When the proper position is obtained and the balance is in equilibrium the root is read off under the cross-hair.

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[1] *Comptes Rendus*, 1840, Vol. 2, pp. 859–60.

As will be noticed the balance will not solve equations of a degree above the fourth and the roots must lie between the practical working limits of about 0.4 to 0.95. However, this limiting of range offers no serious difficulty, for the equation to be solved can always by a suitable transformation $x_1' = kx_1$, $k < 0$ for negative roots, be changed to one whose roots lie within the restricted limits.

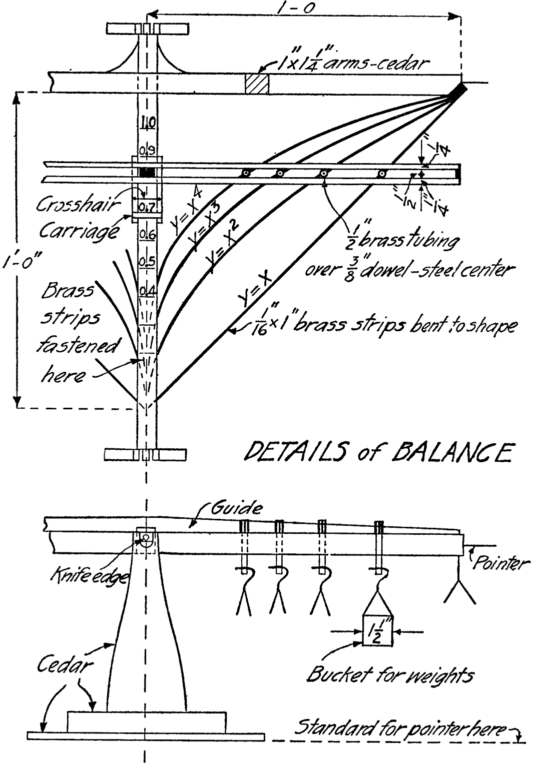

Fig. 1.

For weights 5/16 inch ball bearings (commercial sizes vary by less than 1/10000 of an inch in diameter) serve the purpose well. These are placed in the proper buckets according to the values of the different coefficients. The weight representing a_n is placed in one of the two buckets at the extremities of the balance arms, positive on the right, negative on the left.

The most difficult part of the operation of the balance is to make the stirrups supporting the buckets move freely over the brass curves.

The use of the balance in the solution of an actual problem will illustrate and help fix the principles according to which it works. Let us take one of the equations used in calibrating the balance. The equation[1]

$$x^3 + 3x^2 - 2x - 5 = 0$$

has a root between 1 and 2. Correct to 3 decimal places it is 1.330. This is outside the range of the balance. Form a new equation whose roots are half those of the original. The transformed equation is

$$8x^3 + 12x^2 - 4x - 5 = 0.$$

The root now sought is 0.665. Place the balance in equilibrium with buckets empty. The buckets on the curves $y = \pm x^4$ are left empty. In the bucket on $y = x^3$ (right) are placed 8 of the ball bearings. Similarly $y = x^2$ (right) is loaded with 12. The line $y = -x$ (left) is loaded with 4, and 5 ball bearings are placed in the bucket at the extremity of the left arm of the balance. Equilibrium is again established by means of the guide which is used to move the stirrups supporting the buckets to their proper position. The cross-hair should then be over 0.665 on the scale, the third figure being estimated. Obviously, the root of the original equation is 1.330.

The balance has stood the test of the class-room for a year and can be depended upon for roots correct to two decimal places, with a close approximation to the third. But more than for its worth in solving equations is it to be valued as an aid in fixing in the students' minds quite a number of the fundamental principles in the theory of equations.

GROUPS OF THE FIGURES OF ELEMENTARY GEOMETRY.

By G. A. MILLER, University of Illinois.

The groups of movements of figures of elementary geometry were considered in an article published in this MONTHLY, volume 10 (1903), page 214. The present note is complementary to this article, but the considerations are based upon entirely different principles. In fact, in the present note we inquire into the substitution group, on the sides or the edges of a figure, which transform this figure into others having the same absolute area or the same absolute volume; while the earlier considerations related to those movements which transform the figure as a whole into itself, but interchange some of its parts.

Let P represent any convex plane polygon of n sides. By interchanging two adjacent sides of P without affecting the other sides we can obtain another convex polygon with the same n sides and with the same absolute area. By a succession of such interchanges it is clearly possible to effect a transposition of any two sides without affecting the absolute area. Since every substitution on the n sides is the product of transpositions, it results that the $n!$ polygons obtained from P in the given manner have the same absolute area, and are conjugate under the symmetric group on their sides.

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[1] Fite, *College Algebra*, pp. 192–7.